

## **VERBAL.02 chapter1 expository** Evolution and Language

This is a book about Computational Neuroscience; it is not a book about Evolution, although I suspect she would be displeased to know that. But as with so many things that have to do with brains, neurons, and the science that describes them, there are effects and consequences felt far beyond Neuroscience. Before even defining “computational neuroscience” or “evolution”, I want to violate the most important rule of comedy and start by giving away the punchline of this book: Human language occurred because of an evolutionary mistake so colossal that Darwinian evolution itself is threatened.

Wow. So why did I just say this is not a book about evolution? Let me back up a step. In Science, it is often the unexpected observation that leads to the question that leads to the experiment that tells us something we did not know. Confused yet? The point is that “unexpected”, in its most extreme form, means “Based on what I know, I don’t believe it”. So that is interesting. If I already believe it, then it will not be front-page news. It might make me feel good to confirm something I know; the warm self-satisfied feeling of being correct, but it will not increase my knowledge about the world. Knowledge expands when we learn something we don’t know and the most interesting knowledge expands when we learn something surprising. What I mean is that if you expect one thing, and you observe the opposite of what you expect, then that is interesting. Or if two scientists disagree, and they do an experiment, the one who did not expect the outcome is wrong, but as a result of being wrong learns more than the scientist who expected the result.

Modern experimental science is based on disproving things. You can never prove something is correct. You can just prove that an alternative must be incorrect, usually because it leads to some contradiction. Let me give you an example. Suppose I want to prove that ice is less dense than water. The opposite is that ice is more dense than water. If that were true, an ice cube would sink when placed in water. Try the experiment. Put an ice cube in water. The ice cube does not sink. Therefore it cannot be true that ice is more dense than water. So ice must be less dense than water. If we didn’t know that already, it would be interesting and might lead us to do scientific experiments on the structure of water and ice. We did not expect it because many solid things do sink in liquids, but in this case the solid (ice) did not, and as a result we realize that water, when solid, has some very interesting properties, and people do research on that. In Science, it is the unexpected that moves us forward to new knowledge.

The unexpected thing that drives this book is language. We have it; nothing else does. Yes, your puppy can look into your eyes and “know” how you are feeling; yes, dolphins can communicate between each other and chimpanzees can learn some signs. But it is not the same thing as true language. Just because you can find a few examples of some word-like communication in other species does not mean that language is helping them in even close to the same way it helps us. It is useful to have opposable thumbs and voiceboxes, to live on land where we can farm and make tools and paint on caves, and to have muscles whose primary purpose is facial expression. But we do things with language far beyond what other species do. Dolphins do not write poetry or teach their children mathematics. Chimpanzees do not draw pictures of their family. Your dog may know the sound of your voice and even a few words, but try to tell it a joke. Human Language has many parts to it, including speech, grammar, writing, and extensions into poetry, music, and artistic symbolism. There are many species that communicate, but only humans use recombinations of symbols (words) to form sentences that can explain not only what is, but what we imagine could be.

So back to Science: the unexpected thing about language is that it doesn’t seem to have come from anywhere at all. We have it 100%, but only humans have it. There are no half-

languages or proto-languages. We think that the way things change in species is through Darwinian evolution and genetics: random gene mutations and recombination of genes from your parents may give some members of a species a competitive advantage. Over time, those advantageous genes are passed on, and the advantage more and more favors whoever has the good genes, until everyone has them. But this is a very slow process, and there should be lots of examples of partial language along the way, with competition between people with worse language and those with better language. Except that we don't see this at all. The unexpected observation is the lack of an observation. There is no species that has only nouns. Or only prepositions. That doesn't mean there wasn't some half-language species somewhere, but people have looked, very hard, and not found anything. There are lots of examples of fingers, arms, legs, hair, eyes, kidneys, and skin in other species. But no language.

But once a species has language, it wins. There is no going back. Language is the nuclear weapon of inter-species competition. Not only because language allows you to describe and invent things that don't exist, but because language changes the very nature of evolution. "Evolution" just means that species change over time. The question is why? Charles Darwin's theory in its simplest form suggests that random genetic variation creates lots of opportunities for new traits, and then through competition the animals with better traits out-survive their competition. But before Darwin there were other theories. The theory of Lamarckian evolution is attributed to Sir Jean-Baptiste de Lamarck (a French knight), (shown here in a painting by Charles Tehevenin):

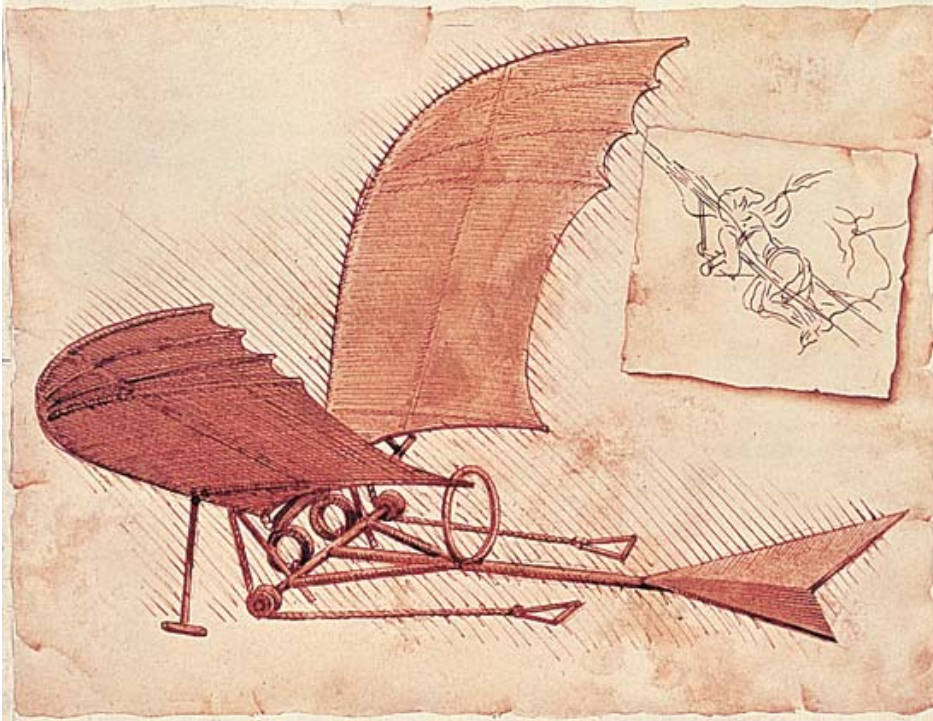


Lamarck believed that acquired traits could be passed from one generation to the next (to be fair, other people had similar ideas, but I'm going to call this "Lamarckian" to keep things simple). If your dad becomes a bodybuilder, you will be stronger. If your mom becomes a university professor, you will be smarter. Darwin would say that you are stronger or smarter because your genes are similar to your dad's and your mom's. In other words, you and your dad are both strong, and you and your mom are both smart, but it's not because of anything your parents did in their lifetime, it's because of the genes they inherited and passed on to you, and it was all completely determined before any of you were born. Lamarck on the other hand, would say you are stronger or smarter because your dad worked hard and your mom studied hard. Well, which is it? The fact is that just by observing human families, it is hard to tell. Because certain traits run in families, which means abilities run in families, which means that children often do things that look similar to their parents.

So it would be helpful to know how traits can be passed on from parents to children, because that is a difference when you know the mechanism of how something happens. Darwin knew something about genetic inheritance, although this was long before the discovery of DNA. But Lamarck did not try to explain how traits could be passed on to children. For instance, how does a beetle that gets stronger by climbing over big rocks pass on its strength to its baby-beetles? How does a chimpanzee who discovers that a particular fruit is poisonous pass on that information to its grandchildren? And in the end, we know that Darwin was (mostly) right; genetic traits are passed through genes and DNA, and the DNA you get from your parents is the DNA they got from their parents, and almost nothing you do during your lifetime has any effect on that DNA (well, there are some exceptions but, like I said, this is a book about Computational Neuroscience, not Genetics). All you can do is mutate DNA, and combine it with a partner to produce children whose DNA is some combination of what the two of you started with. If you lift weights or study hard during your lifetime, the DNA you pass on to your children is the same as if you sat on the couch watching television your whole life.

So Lamarck's theory lacks a mechanism. It simply has no way to work. But what Lamarck's theory does have is speed. Things change in one generation. Darwin has to wait for random variation, competition, selection. And this is happening all the time for many different traits at the same time. Avoiding poisonous fruit will happen only after some random mutation makes some chimpanzees not like the flavor of the poisonous fruit, and only if disliking that flavor does not make them worse in some other way. For Lamarck, it just happens. Done. Like that. As if the chimpanzee could just explain it. If only there were a mechanism to drive Lamarckian inheritance.

Perhaps the Lamarckian mechanism is language. Before there was language, there was only Darwin. The only way to pass on things to your children was through genes. If you learn something or get stronger, your genes don't change, so your children don't get those traits. Lamarck's theory faded out for lack of a mechanism and lack of scientific proof that acquired traits can be passed on. Until language. We now generally believe the knight's theory does not explain the origin of species. Or even the origin of different physical traits within a species. We don't have kidneys because our ancestors tried harder to regulate their blood sodium levels. We have kidneys because things without kidneys (or with worse functioning kidneys) did not survive as long as things that do have kidneys. But what about airplanes? We now have airplanes because our ancestors worked hard to have airplanes. And then they told us about it. Here is a picture drawn by Leonardo da Vinci:



Frustratingly for him, Leonardo never figured out what was important for flight (the shape of the wing's cross-section). But of course others did and they passed on that knowledge by writing it down and teaching it to students. And the wonder of knowledge is that you can pass it from parents to children (if they will listen to you), to other people's children, to other adults, and even to your own parents (who probably already knew it). You can teach better eating habits, better healthcare, how to take care of your heart, your lungs, your bones, and all of those parts of the body work better in future generations. This is all very Lamarckian. It doesn't affect your genes, but it is very fast. Schools and universities and music conservatories and sports teams and companies do it in groups. One person can teach thousands. Books allow us to teach millions. All in less than one generation. Nothing about Darwin going on here. Lamarck is smiling in his knightly grave.

So Darwinian evolution is now in competition with Lamarckian evolution. Who will win? Will Darwin always design a better kidney? Maybe not, if scientists find a medicine to make kidneys work better or to fix ones that break or to eat foods that preserve your kidneys. What about better eyesight? Now we have glasses, telescopes, microscopes, radar, cataract surgery, and retinal lasers. What about strength? Now we have hydraulic machinery, electric motors, jet engines, and anabolic steroids (illegal for sports, but the point is we can do this if we want to). What about social behavior? We have that covered too. Even politics. Competition between species never stops, but Lamarckian evolution always wins on speed. If you learn something, you can pass it on immediately. It is an ability, not really a genetic trait, but does that matter to the woolly mammoth being chased by people with spears? Unfortunate mammoth is not going to evolve armor faster than humans can make sharper spears. Lamarck always wins in a competition with Darwin.

Darwinian evolution depends on "pressure", in the sense that competition for resources (perhaps food) puts pressure on a species so that advantageous mutations are more likely to survive than disadvantageous mutations. But if we remove evolutionary pressure by making warmer coats so that we do not need to evolve fur, or stronger glasses so that we do not need to evolve better eyes, or gasoline engines so that we do not need to evolve faster running, then

there is no pressure for survival of the genetically fittest. You just get a coat, or glasses, or a car, and get on with your life. Your children are not more likely to survive if they are faster runners, because they can just get in the car. So not only does Lamarck win over Darwin, Lamarck stops Darwin in his tracks.

And the first species to figure out how to be Lamarckian can easily stop any other species from getting that ability. Imagine the hippopotamus at the zoo has a baby-hippo that is smarter than all the other hippos. That's nice for baby-hippo; but it will never be promoted to the zookeeper. Even if baby-hippo develops language and starts having its own Lamarckian evolution, humans are at least 200,000 years ahead. And baby hippo does not have a voicebox or opposable thumbs, so even if it figures out the importance of tools it won't be able to make them or brag about them. And keep in mind that humans are very protective of humans. A smarter neanderthal family would not have done well against homo Sapiens once we were talking to each other. We certainly would not give the Neanderthal children the chance to take over from us Sapiens, because we got Lamarcked first and we are holding onto our advantage. Language is the engine of Lamarckian evolution. Words and sentences and books are the "genes" of the new evolution. And skills and knowledge acquired during your lifetime can be passed on to your children. All because of language.

We did it. We got there first. Game over. Homo sapiens has been around less than 200,000 years, and by 35,000 years ago we had wiped out all our significant competitors. In the past 200 years we have grown from a world population of 1 billion to almost 8 billion. While we have nowhere near the population or biomass of beetles, we nevertheless appear to be something new and unique. Beetles, for instance, did not invent bread or the internet. And while many of us still feel that we are losing an ongoing battle with mosquitos, we can usually win against any single mosquito. After all, we have bugspray and screen doors. And more important things to think about.

So that is step one of this book. Once a species has language, it has it forever and it will prevent other species from developing language. That explains why only one species (homo Sapiens) has anything that looks like language. (Sure, maybe dolphins or honeybees have it too, but they are not a threat to us so we don't really care and we don't try to talk to them anyway.) Now all we have to do is figure out where language came from. That will be step two, which is really what this book is about. But step one says that once step two occurs, there is no going back. You can't un-do language. But how do you make it in the first place? How can nouns, verbs, prepositions, dependent clauses, and all the rest of it just appear out of nowhere?

Remember that Darwinian evolution really has two parts: (1) natural selection of the fittest, (2) random changes in heritable genes. Because language is so powerful, it wins immediately on the natural selection part. There is no gradual competition; if you have language, you just win. So now the question is, how does a random change in genes give you language? Language is so complicated that you would think 1000s of genes would have to change in just the right way to make language, and they would have to do all that without messing up other things like your heart valves or your liver function (most genes in your body do more than one thing). The chance that all those genes changed at the same time in just the right way seems very, very, very small. Instead, there should have been lots of small steps toward language as genes changed one at a time and Nature experimented with which ones correctly conjugated irregular verbs. There should have been proto-humans with verb or two, a noun here, an adjective there. Some cave paintings by neanderthals, a badly-rhymed poem by a chimpanzee. But there isn't anything. Only homo sapiens. If you have our genes, you have language. If you don't have our genes, you don't. So how does Charles Darwin explain that? How can you have such a big change all at once with nothing leading up to it. No proto-languages, not even symbolic paintings?

But even for Darwin, things sometimes happen fast. Like when there is a single-gene mutation. Single-gene mutations are all-or-none. You either have it or you don't. Most single-gene mutations kill you or make you very sick. Sickle-cell anemia, Phenylketonuria, Cystic Fibrosis. But what if a single-gene mutation made you better? A lot better? Even gave you language? How silly (you say), language is way too complicated to be a single-gene mutation. But is it?

Punchline spoiler: I propose that a single-gene mutation can cause language.

What I am going to propose is that the single-gene mutation that did it is not about language, it is about symbols. Symbolic thought to be precise. The main thing about language is that it depends on symbols. Words are not interesting except that they talk about things in the world. Verbs are not interesting except that they say how things move and change and act. The word "if" is not interesting except that it allows you to imagine things that could be. Cave paintings are just marks on rock except that they stand for animals and events you have seen. So the first thing you need is symbols. It's also helpful to have grammar, and lots of people have studied grammar, and I'm not going to talk about that at all, except to conjecture that once you have symbols for things and symbols for actions you can (and will) put them together into thing-action pairs and we call that grammar, and grammar itself evolves according to Lamarck as we teach more complicated concepts to our children.

But without symbols you cannot have language. Words stand for something. And they do so very imperfectly. The word "tree" is a category, but there are things that are almost trees, things that are pictures of trees, and things that might have been trees but are actually bushes. But the imperfection allows us to use logic to talk about trees, and to imagine purple trees, or upside-down trees, or metaphorical trees such as the Tree of Life. Symbols are very different from perception, because when we see something we see that particular thing, warts and all, and we can decide how to react to it. But when we use a word we are talking about all things in the category and we can learn how to react to one tree based on how we have reacted to other trees. It's all very powerful. And very different from non-symbolic thought.

So symbols are not just about communication, but about thought. You have to have symbolic thought, otherwise language doesn't work. If you don't know what a frog is, there is no point in saying the word "frog". And if someone else says "frog" you have to convert the sound into the symbol in your head. Whoa, you say, this is getting far too metaphysical for me. OK, that's fine. I'm going to explain all of this in the rest of the book. For now, stay with me on the following ideas: (1) language depends on words, (2) words are symbols, (3) symbols require symbolic thought.

So could symbols happen as a single-gene mutation? I think so, and that is where we come back to Computational Neuroscience. Computational Neuroscience uses Math to understand brains. Which is very circular, because Math is invented by brains. Yes indeed. Animals don't do math. Math symbols do not appear in the clouds or in the ocean. We invented Math, because it is useful to us. And (please don't be scared) there is Math in this book, although I will explain everything as we go. But Math is important here, because it provides an answer to the question: what if, to Computational Neuroscience, the difference between symbolic thought and non-symbolic thought is very simple? What if they are two sides of the same thing, like water and ice. Water and ice seem very different, but you can get from one to the other just by changing the temperature one degree. So what if some gene that was making better and better water suddenly went too far and made ice? Or what if some gene that was making better and better non-symbolic thought suddenly went too far and made symbols? Then a single gene and a very slight mutation could be the difference between language and no language.

That is really what the rest of this book is about. How a very simple change in a single value in the brain could create symbolic thought, which leads to words, which leads to language. Here's the answer, in case you feel like skipping the rest of the book: symbols are binary in the sense of a computer having binary bits that are either 0 or 1 but nothing else. You either say the word "frog" or you don't. Whether or not it actually is a frog is not the point. You either said the word or you didn't. Saying it quietly does not change the fact that you said "frog". But the real world is not binary and it does not use bits. Maybe you are looking at a toad. Or a hamster painted green. So we evolved for a non-binary (continuous) world of continuous variation, but we use binary words to describe it. Talk about a square peg in a round hole. Should have failed miserably. But it didn't. That's why Evolution is so angry. And later in this book, I'm going to show how just by turning up the volume on nerve cells you can make them go from continuous to binary. That's it. Just turn up the volume. Way too loud. So that all you can really hear is whether it is on or off. Voila! Binary. The genetic mistake that changed the world. So maybe, just maybe, a single gene mutation that turned the volume up in just the right parts of the brain forced binary logic which forced symbols which allowed words which led to language which then developed grammar using Lamarckian evolution. Lots of steps. I'm really only going to discuss the first one.

I suppose the same single-gene mutation could have happened in other species many times during evolution. Symbolic thought would not be too helpful in horses or fish or maybe even dolphins, but what if it happened in a species that had already evolved fingers and opposable thumbs (originally for feeding) and a voicebox (originally for making loud noises when a threat appears)? Now the brain has symbols, and the body has fingers and voice. A symbol turned into sound is a word. A symbol turned into a finger movement (if your fingers are dirty) is a shape drawn in a smudge on the rock wall. Sure, you can still make facial expressions that your cat will understand (even if she pretends she doesn't), but you can also draw a map to where food can be found, or a picture of a dangerous animal, or use a particular smudge to indicate a person so that you can say how many people there are in the neighboring tribe of Neanderthals.

So maybe a single-gene mutation could do all that. Just one mistake in making a copy of just the wrong (right?) gene. Maybe it could happen in one generation. Maybe it only mattered in a species that had fingers (and to use fingers properly, it is also helpful if you walk on two legs) and voiceboxes. If we waited long enough, Darwin would say that the same mutation could happen in another species with opposable thumbs, perhaps a Neanderthal, or a Koala bear. But Homo Sapiens wasn't going to wait around for that. We wiped out all the neanderthals before they could mutate. And Koalas sleep most of the day. It's not that the early humans were thinking that mutations might happen, they just wiped out any real competition because they could. And very quickly too. Because if you can teach your child what is poisonous, you can teach it to use poison. You don't have to actually poison anything to show the principle; you can just talk about the possibility. If you can teach your child to avoid sharp objects, you can teach it to throw those sharp objects at your enemies. And you can explain the concept of "sharp". If you can see that fire can be carried from one place to another on a burning piece of wood, you can teach your child to use fire and not to be afraid of it even if your enemies are afraid of it. And you can teach the concept of "hot". The other species all are waiting for Darwin's evolution to give them a chance to compete, but we have just changed the rules of the entire game of evolution. All the other species are fighting with fists while we have airplanes. Doesn't matter how good your fists are. We play by Lamarck's rules while the other species play by Darwin's rules. It's not a fair fight. Language changed the game and there is no going back. The irony of course, is that Darwinian evolution led to the genetic mutation that led to the end of Darwinian evolution (at least for homo Sapiens).

So I propose that a single mutation is enough to give you symbolic thought which is the key to all the beautiful glory of language: all the creativity, all the imagination, all the creativity, all the

communication, all the understanding. And the route to this possibility lies in Computational Neuroscience. Because there is a way. I can't prove it, and I won't. I don't know the name of the gene that did this. This is just a theory. A theory that says this is possible. It could possibly have happened this way. It's probably not the only way. It's just one possibility. But it is a real possibility. Theories are always in competition with each other, and the one I propose is no exception. Some future theory will compete with it, and one or both might win. This theory might even provide the basis to disprove itself. In Science, that is ok. The point is to move things forward. And sometimes things circle back with new understanding. Lamarck thought he had the answer until Darwin showed everyone a better theory. But it is not impossible that Darwin's evolution led to a mutation that allowed Lamarck's evolution to win. My goal is to explain how that could have happened.



## VERBAL.03 chapter2 dialogue

### “Two Worlds”

Goat: Good morning everyone! Welcome to the first day of your adult education build-a-brain class. I'm pleased that most of you are on time and have found your seats. This will be an unusual class, and I'm not referring to the fact that you are all talking animals. What makes this class unlike other build-a-brain classes is that we have a purpose. We don't want just any old brain that thinks, walks, smiles, or learns to juggle. not us. We want a brain that talks. That's useful because, quite frankly, a brain that doesn't talk is just a blob of jello on the desk and no one likes jello on their desk. Those of you enrolled in build-an-ear and build-a-voicebox classes last semester are welcome to use your final projects from those classes to help the brain communicate with us. The whole point is that unless we can talk to it, how can we know what it is thinking? It could be making fun of my bowtie (in fact, don't ask it. I don't want to know. I'm proud of this bowtie. Special-order plaid.)

Ok, so that's the plan. Build a brain that talks. Or at least a brain that can make a voicebox talk when it wants to. To be quite honest, I've never done this before. I hope it's possible. If not, then you all fail! really. I will fail you. So you need to get this done. I suggest you all work as a team. Or not. Your choice. But I don't think this is easy. It's really only been done once before, by Evolution, and we have no idea how she did it. We don't even really know what it is that evolution did. So Good Luck!

First step, let's all introduce ourselves. I'll start. My name is Bill. I'm a goat. An old one. I'm not here to make your lives miserable, but I'm willing to try. At least I'm honest about that. Try that with Jim who is teaching build-a-unicorn in the next class over. Ha! Ok, you first.

Unicorn: Well aren't you the perfect gentleman Mr. Bill Goat. I am certain that you and I and this delightful class are going to have a truly exceptional semester together. My name is Constance. As you can see, I'm a unicorn. And a lady. So I insist on proper behavior from all the less fortunate un-horned species here, and their creations. So sorry to be such a bore about this, but there really is not a purpose to building a brain if it does not have good manners.

Goat: Point taken.

Unicorn: Oh sweetness; don't take any points from me. I always have the point, and I'm not giving it to you. So. I would like everyone here to know that I am a particular specialist in rainbows. I love rainbows, I admire rainbows, I talk about rainbows, and I know most everything about them. Maybe you read my book on the great rainbow of '75? I hope so. It was spectacular, with some truly colorful characters. Rainbows have all the true colors of Nature. I trust that all you beautiful people here believe that.

Elephant: What about Magenta?

Unicorn: Don't go there.

Elephant: I just went there. I don't see Magenta in your rainbow. Magenta is my favorite color.

Lizard: What about White?

Constance/Unicorn: You are too green to be offering advice on color. But I'll forgive both of you for your explicable ignorance, because you seem so eager to learn, and I'm here to enlighten you. Magenta and White are not part of the rainbow. They are mixes of colors. Like the color you get when you mix all the paints together. And mixing colors makes ugly colors. Not pure and beautiful like my rainbow. In time, you will come to learn the beautiful colors.

And speaking of colors; I think that it is very important that our build-a-brain be able to talk about different colors properly. Thank you all for listening.

Elephant: Well now that Ms. Constance is done with constant talking, Let me tell you a little about myself. My name is Ned. Everyone calls me Ned. It's not short for Nedward, or Nedtastic, or Alphaned. Just Ned. And I gotta tell you, I hate Nuance. So you can call me No-Nuance-Ned. For example, I'm an elephant. You know that because I have a trunk. No trunk: no elephant. Got trunk: elephant. Got it ? Simple. No nuance.

Owl: But trees have trunks. You could be a tree. Or a car.

Ned/Elephant: Don't go splitting feathers on me. Trees don't have legs.

Owl: So you need a trunk and legs? I once saw a horse bringing a large trunk of clothes on vacation. The horse had legs, and it also had a trunk.

Ned/Elephant: Aagh! You're nuancing me. I'm an elephant because I have a trunk and I have legs, and my trunk is my nose, and there's no tree in this room, and there's no horse with a steamer-trunk in this room.

Owl: Ok, Ok. You don't have to get all snorty about it. How about we agree that here in this room, if you have a trunk you are an elephant. Once you are outside of this room, if you have a trunk you can be anything you want.

Ned/Elephant: I don't want to be anything I want. I am an elephant. period. done. Just like you are an owl.

Owl: Spotted Owl.

Ned/Elephant: Still an Owl.

Owl: Spotted. Spots are part of my identity. Just as, it appears, a trunk is part of yours. I hate to nuancify you, but have you ever thought about what would happen if you misplaced your trunk? Would you still be an elephant by your definition?

Ned/Elephant: I'm always an elephant. And what about Ms. Unicorn? Her very name says that she is defined by having one horn.

Owl: Just like the jazz band I was listening to last night. One horn, one guitar, one piano, one bass. Didn't look like a unicorn.

Constance/Unicorn: Neither does my car, and it has one horn too. And a trunk. I will leave it to Ned to un-nuance this annoyance.

Ned/Elephant: well I can't define a unicorn, or an elephant, but I know one when I see one. And that says something because I've never seen a unicorn before.

Owl: If you didn't know what a unicorn was, I might have to show you a thousand pictures of unicorns so you would get the idea. But of course, one word is worth a thousand pictures. And one horn tells you all you need to know.

Bill/Goat: I'm glad we are all getting to know each other. Owl, since you had so much to say, why don't you tell us about yourself?

Owl: Well, it appears I am probably the most erudite amongst those present, begging your estimable pardon. My name is Who, which is the unanswerable question you just refrained from verbalizing. While Ms. Constance may be an expert in rainbows, I fancy myself an expert in the arts of logic and reasoning. My tools are words, my method is logic and my object is truth. I look forward to contributing my perhaps excessively ample knowledge of such things to our shared project constructing a talking brain. I expect that if you follow my guidance, this should be a straightforward project. Words after all, are the fundamental medium of logic, which is essential for rational thought.

Constance/Unicorn: Perhaps, but there is no beauty in Logic.

Who/Owl: ahhhh but there is beauty in Logic. I am very logical. I am also very beautiful. Therefore logic is beautiful. Some might call it Sophistry but I would counter that Sophistry is the root of Sophistication and is thus Sufficient for rationality. You will see in time, if you pay attention.

Bill/Goat: How about you, Lizard?

Lizard: I just need this class to graduate. Don't really care much for brains. I mostly catch flies. If you can help me make a brain that catches flies I'm all for it. Even a brain that helps me catch flies. And I also like poetry, particularly when it flies in the face of reason. Unless reason eats it. Anyway, you can call me Leonardo. I was named after a great artist. Which explains why I also like art. But that's not my real name; my real name is unpronounceable. So I suggest you all use Leonardo. And leave the screens open please. Unless you have fleas. Or are trying to sneeze. Or there is too much of a breeze. Thanks.

Bill/Goat: And you at the back; you came in late so I didn't see you. Tell us about yourself?

Cat (yawning): My name is Claude. My parents named me after Claude Shannon but really I think they kept the name because I like to scratch the furniture. I'm really good at that. I also like to bite things. Not any of you; just bite. That was a thing with the other Claude I think. Lots of bites. Mostly he liked getting information from one place to another, which is kind of what I do since I'm a bicycle messenger when I'm not going to school. I guess I want to build a brain that gets information from one place to another. That might save me some work. And I'm already good at that so if you all listen to me, I think we can do this. Not so good with rainbows but Ms. Constantly here can be somewhere over that department. I don't know what a nuance is but I'm happy to find out if it doesn't make Ned angry. And Mr. Who; I don't even know what an Erudite is, although quite frankly if I met one I would cross to the other side of the street; my mother always told me not to be rude, e-rude, or rude-ite so we got some interpersonal working to do here I think. Stay with me brother-who and we will make it work. Now back to my nap.

Bill/Goat: OK, now that we have that all sorted out (and no sleeping in class Claude), we should get started. First question: what do we need to make a brain talk?

Who/Owl: ooh ooh ooh I know! We need language.

Bill/Goat: Of course, but what is language?

Leonardo/Lizard: Language is words. Although I'm also a big fan of body language. Especially a nice small buzzing body I can eat. Very sweet.

Bill/Goat: Words are just a type of sound. What makes a word a word?

Ned/Elephant: You love to nuance me don't you? A word is a word. Sound that means something. You can tell people something with it.

Who/Owl: Ah yes, it always comes down to meaning doesn't it? But what do you mean? Consider my very own name: "Who" could just mean it's own sound, like a "who" noise. Or it could mean me. Or it could be a question about who understands me, which refers to a group of people who understand me (which is fortunate for them).

Ned/Elephant: Don't much matter to me which one you mean, so long as you are clear about it. Let's take an easier word. How about "blue".

Constance/Unicorn: Ah, now you are talking my language. I'm thinking of the blue of the sky two weeks ago over west candyland at around 3:15 in the afternoon. What a color. It matched my mother's eyes.

Ned/Elephant: I'm sure your mother has beautiful eyes. But any old blue will do.

Constance/Unicorn: No "any old blue" most definitely will not do. There is no such thing as "any old blue". The rainbow has every color, and lumping groups of colors together is disrespectful. My mother's eyes have nothing to do with the color of the sky at 3:17 that afternoon. Different entirely. If your so-called "words" tell me those are the same then I say phooey to your idea of language. Phooey. There, I said it. To your idea of language. Harumph. Fiddlesticks.

Claude/Cat: (yawning noisily): But you can't put your mother's eyes in an envelope for me to messenger across town. How do you tell the drapery store what color you want? And supposing (just for the sake of argument) that you send a picture of your mother's eyes, but (maybe) I take a long time to ride across town because it's hot and the color fades and then (perhaps) I stop off to claw some furniture on the way, and then (possibly) I drop your picture in a very brown puddle of mud and (probably) when it gets to the drapery store they think your mother's eyes are a nice shade of magenta-brown. If you had instead just written the word "blue" on a piece of paper, even on a hot day and dropped into mud they could still tell that you wanted something blue.

Leonardo/Lizard: gotta point there bubs. But a picture is worth a thousand words. I like to paint. I can paint something without using any words at all. And you know what it is just by looking. Same for a color; I can take a picture of Contance's mother's eyes and send that in a waterproof bag. Don't need a word or anything.

Who/Owl: Well, if you will permit me to split a hair or two, what if you just paint the word "blue"? Or more interestingly, use blue paint to paint the word "red"? Does the blue/red word mean red or blue?

Ned/Elephant: depends on whether you decide to read it or look at it. Personally, I would just look at it.

Claude/Cat: Have you ever tried? Even without nuance, I bet you have a hard time seeing the picture for the words. Try it.... I doodled a page full of words written in different colors. Try to just say the colors as fast as you can without reading the words.

yellow blue green blue  
yellow red blue red  
green red yellow yellow  
blue yellow yellow green  
blue green green red  
blue red blue yellow

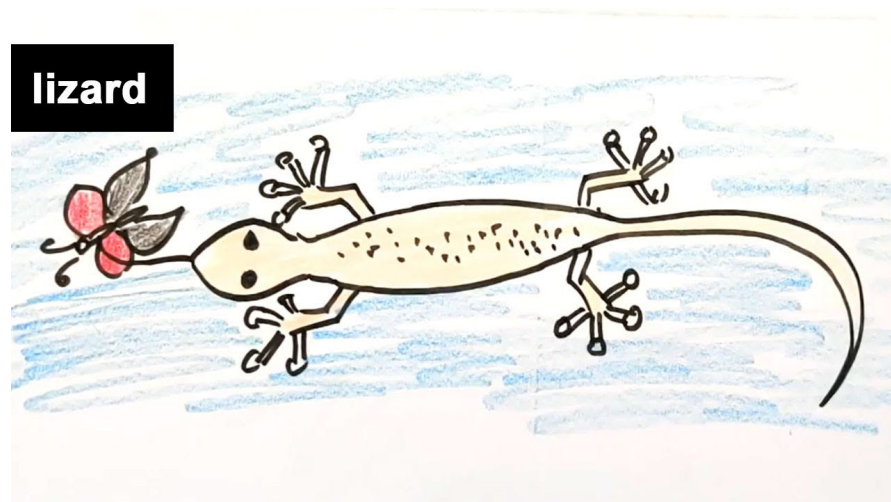
Ned/Elephant: blue... green ..... green.... red... yellow... aaaaugh! That's really annoying.

Claude/Cat: Yup. Guy named John Ridley Stroop invented that. Has been annoying people for almost 100 years.

Who/Owl: You see my dear fellow; you are wired for words.

Constance/Unicorn: which is a shame, because they don't serve much purpose. None of Mr. Stroop's words reflect the precise beauty of a rainbow.

Leonardo/Lizard: I have an idea: why couldn't we ask someone who did well in build-a-mouth class to make something that speaks pictures instead of words? Here's my doodle: a picture of a beautiful lizard about to eat a yummy bug.



Claude/Cat: That's just weird. What do you think this picture tells you Ned?

Ned/Elephant: I see a bug, and a lizard. Don't know which Leo is trying to tell me about. Is this supposed to be Leonardo? Or any old lizard? Or Leonardo's lunch today? Or any old lunch? Or maybe it's just about something eating something else.

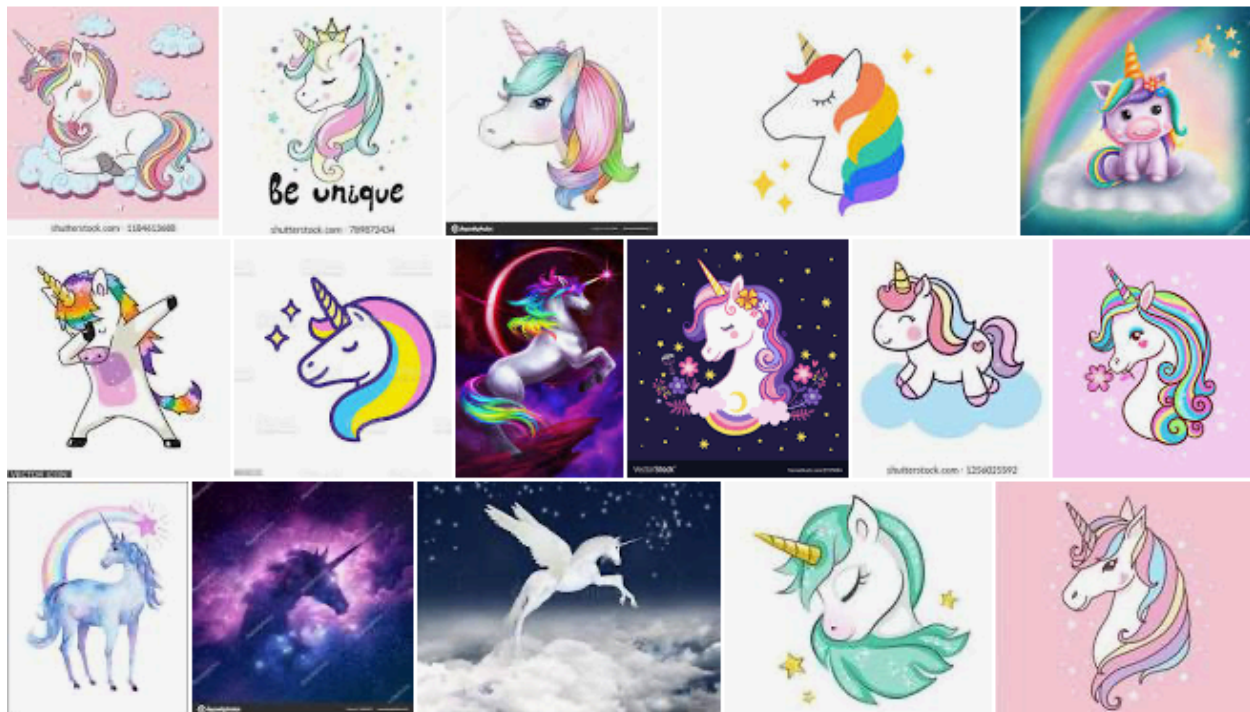
Leonardo/Lizard: OK, I'll draw you another one. How about that?

Ned/Elephant: Oh I see; you're trying to tell me about Lunch! Is anyone else hungry?

Leonardo/Lizard: No, no, no!!! This isn't about lunch, it's about a lizard (me actually).

Ned/Elephant: Well how would I know that? Maybe a picture is worth a thousand words, but you can't tell which ones. Try to figure out which 1000 words Picasso had in mind.

Constance/Unicorn: Sounds like you are nuancing now Ned. But here's a better example: a picture of the 16 most beautiful unicorns (I'm one of them).



Leonardo/Lizard: Yup. Lotta unicorns.

Constance/Unicorn: No, no no!!! That's not the point. Each of us is unique. Once you call it just "unicorn" you lump us all together like a large pile of pointy heads. Don't you see these are unique-horns?

Who/Owl: Multiple unicorns. But at least I could figure out what you mean by the word "unicorn" if you told me these were all the same thing.

Constance/Unicorn: hopeless. Once you use the word, you remove the magic.

Leonardo/Lizard: like a poetry critic.

Who/Owl: I prefer words. I can easily say determine unicorn is which, by precisely saying “the unicorn with the blue eyes and the long tail”, or “the unicorn in the top left corner”, or “the unicorn with two horns”.

Constance/Unicorn: words, words words. No magic. No art.

Leonardo/Lizard: I write poetry. There once was a tadpole from Shropshire...

Ned/Elephant: I don't want to know where this is going.

Claude/Cat: The point is, words can be beautiful. Consider “cuspidor”, one of James Joyce's favorite words. Or Bertrand Russell who likes “ineluctable”. Or perhaps ineluctability.

Leonardo/Lizard: Cuspidor might be beautiful, but a spittoon is not. I can't un-think that image. I wish I could chew on a bug right now.

Constance/Unicorn: words are beautiful if you can make a picture out of them. Consider “The rainbow warmed the sad bird's heart.” Except for “rainbow”, none of those words has much beauty, but I like the picture. Maybe Leonardo could draw that.

Leonardo/Lizard: I don't think I could make a picture say exactly that. Unless maybe I do a comic. I'm seeing a bird heart on a block of ice, then a rainbow walks up and lights a fire under the ice, which melts, and then the fire warms the bird heart. Maybe the bird is nearby being sad about all this. I dunno.

Constance/Unicorn: That is definitely not what I meant. Right words, wrong picture.

Who/Owl: You see? There are some things that need words.

Ned/Elephant: But words can be weird. What about “Time flies like an arrow, but fruit flies like a banana”? Same words, different stuff. Makes my no-nuanced skin crawl just like magenta does to Ms. Constance.

Leonardo/Lizard: I'm willing to bet neither you nor your skin know how to crawl.

Who/Owl: Sometimes even writing things down can be confusing. Consider the roman numeral “IX” which indicates “9” which we could also write as “oooooooo” if we were talking about nine o's. A young goat once asked me if I could add a single line to the roman numeral IX and get “6”. Of course you can't, which is one of the things I don't like about goats. And Romans.

Leonardo/Lizard: What about “SIX”?

Who/Owl: Ugh. I h8 w1 people 4ce me 2 use numbers duplicitously. Makes me mad and when I'm mad I hoot at people. Don't make me hoot. You'd be an \* it.

Ned/Elephant: Those are too many abbreviations. I don't like to abbreviate.

Constance/Unicorn: But certainly, even an elephant can see that words are very imprecise; each word can mean so many things, and each thing can be described by so many words.

Claude/Cat: But certainly, even a unicorn can see that once you say a word, that word is said. If I say “blue”, then the word “blue” has been said. If you paint a picture of blue, the color itself could change. But the word does not change. Said is said. Done is done. That's precise.

Constance/unicorn: What does it matter if the word is said precisely but the thing it means is not precise?

Ned/Elephant: Seems to me at least the word itself is not nuanced. You can choose to make it mean something weird, but I know what I meant when I said it. Said is said.

Constance/Unicorn: But you cannot use words to describe the beauty of my mother's eyes.

Ned/Elephant. Sure I can. "The color of your mother's eyes." There. I did it. Exactly.

Constance/Unicorn: That's cheating.

Who/Owl: I think Elephant has a point.

Constance/Unicorn: No, he is quite round. I have a point.

Claude/Cat: You both have different points. But the point is that you both said "point". You didn't have to say it, but you did. Even if you meant something different, I know exactly what you said. And I could write it down and carry it across town if someone points me the way.

Leonardo/Lizard: Seems to me like the same word can mean many different things. That's good for poetry. I think that I shall never see a poem as pointed as a tree.

Constance/Unicorn: I don't like that words mean so many things. There are infinitely many beautiful colors of blue in the word "blue".

Claude/Cat: Even if you write the word "blue" using black ink.

Who/Owl: But increasing precision is possible. Suppose I tell you that my acquaintance Howl the Owl made a loud hoot. You know the hoot was louder than if I said it was not a loud hoot.

Ned/Elephant: Yup. Loud hoot. I hate loud hoot.

Leonardo/Lizard: But exactly how loud was it?

Who/Owl: Very loud.

Leonardo/Lizard: Not working for me. I couldn't make a hoot that was just as loud just because you said "very loud".

Who/Owl: That's not the point. I don't care how loud you can hoot. I'm trying to tell you something about Howl the Owl and his very loud hoot.

Constance/Unicorn: I don't see your point.

Claude/Cat: Owl's point is that unless you plan on strapping Howl the Owl to the back of my bicycle, at least I can bring a message about the loudness of his hoot across town, even if you don't know exactly how loud the hoot was.

Leonardo/Lizard: I don't give a hoot, loud or otherwise.

Who/Owl: You would if you heard Howl's hoot. It's very loud.



Constance/Unicorn: So you said.

Leonardo/Lizard: But there are many loud hoots that could all be called “very loud”.

Claude/Cat: So what? At least you know it wasn’t a quiet hoot. That’s something.

Leonardo/Lizard: It’s not much.

Claude/Cat: Better than nothing. And you could add more words. Like “extremely very loud”, or “louder than a siren”, or “ear-splittingly loud”.

Who/Owl: And Howl is loud. Although splitting an ear is not something that he has ever done intentionally. But he is loud.

Ned/Elephant: I’m getting that.

Who/Owl: So you learned something. Even though Howl isn’t here. Actually, I made him up. I do not have an acquaintance named Howl. It’s just a hypothetical.

Constance/Unicorn: No point to it.

Leonardo/Lizard: A bit like poetry. You can be creative with words like “very”.

Ned/Elephant: Seems to me that you need words like “loud” and “hoot” to talk about things that didn’t happen. Because you can’t actually listen to them. Because they didn’t happen. But you can still talk about them.

Constance/Unicorn: You are repeating yourself.

Ned/Elephant: And you are talking about me repeating myself. Hah! Imitation is like flattery.

Who/Owl: Not always. I have another acquaintance who is a mime. Not flattering.

Leonardo/Lizard: Is this a real or a made-up acquaintance?

Who/Owl: Doesn’t matter. I can use my words to make things up in order to prove my point.

Constance/Unicorn: Which you don’t have anyway.

Who/Owl: At least I have imagination.

Leonardo/Lizard: But you are not particularly imaginative, even if your words are imaginary. Although I agree that words do let you describe things that didn’t happen. Or that could happen. Or that you want to happen. Or that someone else told you might have happened to their distant uncle.

Bill/Goat: All right, that’s enough of the pleasantries. You are all so enthusiastic; warms the mussels of my heart. Also the cockles of my heart. All very interesting, very interesting I’m sure. But if you want to pass this class, you will have to build me a talking brain.

Leonardo/Lizard: That’s where I disagree. We need to build a communicating brain. Who says talking is so wonderful. Maybe it does mouth-painting. I say we have two teams and we race. Constance and I will be the team of beauty and magic and poetry, and the rest of you all, Ned, Claude, and Who can be the team of boring words with boring meanings.

Who/Owl: I take immediate offense at being called boring; Boring is only useful for making tunnels. I would much rather be pedantic, verbose, or loquacious. But on behalf of my less precise colleagues, I accept the challenge.

Bill/Goat: Then so shall it be. I shall call you “team Unicorn” and “team Owl” for now, until you find better names, or perhaps pictures, for yourselves and your brains and your robots.

Leonardo/Lizard: I thought of a poem to commemorate the start of this class:

The Owl and the Unicorn shall compete; in a classroom taught by goat.  
They argue if words and a couple of verbs will suffice for a messenger’s note.  
They toil away for a year and a day or as long as the semester lasts,  
To see which ‘bot, verbal or not, will win the prize for the class, the class,  
will win the prize for the class.

## VERBAL.04 chapter2 expository

### Discrete and Continuous

The Owl and the Unicorn represent two different worlds of communication. The Owl lives in a world of words and language, in which words have meaning and beauty on their own. Words can be used to communicate concepts, make logical connections, and describe observations and behavior in the world. As Elephant learned, words can have multiple meanings, but that is part of their power. Words can also create great specificity, allowing one to determine exactly which unicorn is being discussed. While words can have multiple meanings, they are not wildly ambiguous; if you see a lizard, it is correctly called a lizard (among perhaps other names). If I tell you to imagine a lizard, you may have a very different picture in your head than I do, but that doesn't matter because we are both imagining things that could probably be called lizards.

The problem is that the word "lizard" does not indicate a particular lizard, or even a particular type of lizard. No matter how many words you use, you can never really drill down to an exact description. For example, "a green and white spotted lizard that studied harpsichord composition" still describes a group of potential lizards, not a specific one. Even "Leonardo the Lizard" is a category that could be filled by any lizard with the name Leonardo. A picture, on the other hand, is a picture of a particular lizard. It may not be one that has ever existed, but it can be photographically precise. There is probably only one thing (or no thing at all) that matches the picture. These distinctions are not perfect; "the lizard in this room" specifies only one lizard, and a cartoon drawing of a lizard could specify many lizards. But the point is that words and pictures commonly do different things.

The difference is that a word creates a *category*; a group of things that is consistent with that word, and a group of things that isn't. This is the case, even when the category is itself somewhat ambiguous. For example, the word "trunk" could represent an elephant trunk, a tree trunk, or a steamer trunk, but it cannot represent a tomato. Owl, Cat, and Elephant all support the use of words, for different reasons. Owl supports words because she believes in logic and reasoning, which are things that words are particularly good at. Cat supports words because he uses them to transmit messages on his bicycle messenger route. And Elephant supports words because he believes (sometimes incorrectly) that they are unambiguous and have only a single or small number of meanings.

Lizard and Unicorn take a different approach, that is in some ways more precise. Unicorn believes that each color of the rainbow is a different color; if you group colors together in a category such as "blue" you lose the subtlety and the precision of each of the different types of blue. If you group a set of pictures of unicorns together, you lose the individuality that makes them special. Lizard likes to paint, and the painting itself is the unique thing being communicated. You cannot reconstruct a painting from words, and there are many different sets of words that can describe each painting. The same set of words can describe many different paintings, such as a painting of "heartwarming".

That is not to say that pictures cannot be used as words. Pictographic languages such as ancient Egyptian or modern Chinese use small icons that in some cases look like a picture of the thing they are describing. For instance, here is a segment of Hieroglyphic writing from the Rosetta stone:



The pictures do not necessarily represent the thing they are a picture of. For example, a picture of a bird could represent a bird, or the “b-“ sound (if this were English). A very similar system exists in modern Japanese, in which a picture can represent the sound of the Chinese word for the picture (early Japanese writing was derived from Chinese writing) or the sound (and meaning) of the Japanese word for the picture. For example, suppose France used a pictorial system in which the emoji :( was used for the word “sad”, written in French as “triste”. If we brought this to English and used it as a letter, we could use the :( symbol to indicate the “tr-“ sound (first part of the French word) or the “sa-“ sound (first part of the English word) or the word “sad”. The point is, none of this is anything like what Lizard was proposing: communicating using pictures instead of words. The written words (and the spoken language that writing converts to) are all categories (of sounds, or of emotions), whereas a picture is not a category. It is a thing in and of itself.

What I am getting to is the idea that words are *Symbols*. A word represents a category of things. In contrast, a picture represents exactly one thing, although it could be a member of a category. There is a gray zone: what about a cartoon or an emoji? What about a painting of a particular person, or a group of people, or many unicorns? Yes of course. There is always some fuzziness to be found if you look for it. But the point is that Symbols are not Representations.

Representations (pictures, sounds, art, music) are very close to reality. A good enough picture (with 3D glasses on) may be indistinguishable from reality, in the sense that our experience of the picture is similar to our experience of reality. The intent is that our brain reacts the same way to both, or at least approximately. The cubists thought that a distorted version of reality could create a similar experience to true reality, although it is hard to imagine confusing one with the other.



Both pictures show the same scene, but neither picture is a Symbol because it is a particular woman holding a particular teacup. I leave it up to you to determine if Jean Metzinger's painting is more real than reality. But the point is that representations try to create a particular image, sound, feeling, or thought in our brains that is somehow similar to something we might actually have seen, heard, felt, or thought.

A mockingbird that mimics the sound of a pigeon is not trying to tell its friend "hey, check out the cute pigeon over there." It is just trying to keep other pigeons out of its territory, or perhaps impress a potential mate with its comic imitations. A parrot that learns to say "Polly wants a cracker" does not necessarily want a cracker, or even know that its name is Polly, although it may notice that when it makes this particular noise it sometimes gets dry food. These birds are mimicking reality for a purpose. It is a form of communication, but it is not Symbolic communication.

So which is more precise; Symbolic or non-Symbolic? That depends on how you look at it. If I tell you  $2+2=4$ , that is a true statement. Two plus two is exactly four. Always, forever. This kind of thing makes Ned happy. But that is because the symbols "2" and "4" are living within their own symbolic world. As soon as they have to interact with the real world, everything goes all pear-shaped (I love that expression; it's British.) Suppose I measure two cups of sugar and two cups of flour and mix them together. Do I get four cups of powder? Well, approximately. Because of course my measuring cups are not perfect so I probably didn't have exactly two cups of sugar and flour to begin with. No problem, but then what do we mean by  $2+2=4$  when we are cooking? Approximately two plus approximately two equals approximately four? OK, but now "approximately two" is a category that includes everything from 1.9 to 2.1. Or, if you are messier than me, maybe 1.8 to 2.2. Or if your measuring cup runneth over at 2 cups, then 1.8 to 2.0. As soon as you start interacting with reality, then Symbols become only approximate. That doesn't mean they aren't useful (my pound cake still tastes excellent, in my opinion).

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Goat's linguistically accurate Pound Cake Recipe:

approximately 1lb butter

approximately 1lb eggs

approximately 1lb sugar

approximately 1lb flour

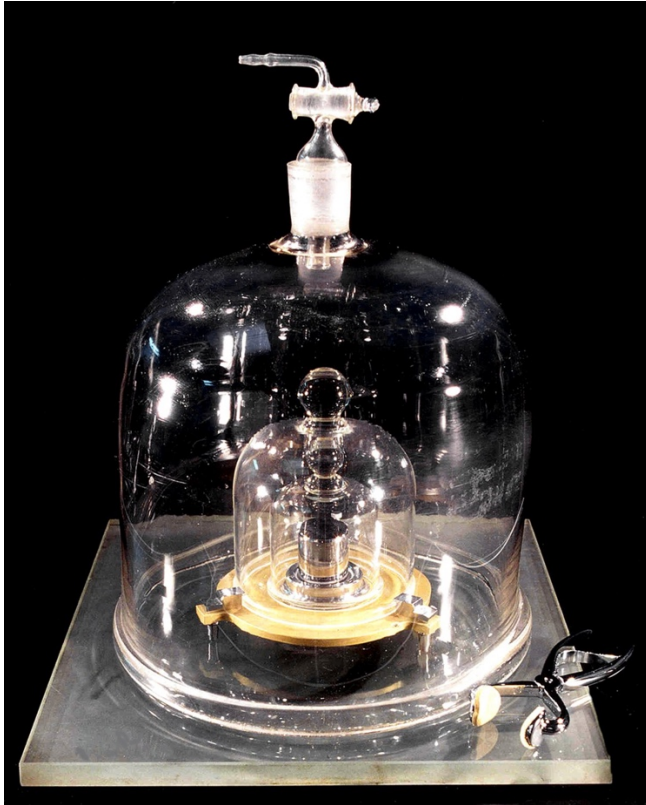
more or less 4lb total

cook for about the amount of time required to drive from Manhattan to Pound Ridge.

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Symbols are perfect in the world of symbols. So what about non-symbols? Non-symbols can also be truly perfect. When the Bureau of Standards decides how heavy is a pound, they don't say it is equal to 16 ounces, or 0.45359237 kilograms (which is true, but circular reasoning).

They actually have something that weighs a pound. At least they used to. There is something that weighs a kilogram because it defines a kilogram. One thing, carefully protected in Paris, that is the perfect kilogram (at least until 2018 when the scientists got tired of traveling to France to measure ingredients for kilogram cake and came up with a fancier way to do this).

Just the way Unicorn's mother's eyes have a particular color that is defined only because it is the color of her eyes. Here is a picture of exactly one kilogram:



Of course, if someone hands you the perfect kilogram and you use it to weigh out some flour for your kilogram cake, your scale may not be perfectly accurate, so you will never get an *exact* kilogram. But that doesn't matter; the point is that there is, somewhere, an exact kilogram. Just because you can't get the color of the drapes to match your mother's eyes does not mean her eyes don't have a particular color. And, really, nothing else has *exactly* that color, just as there are not two *exactly identical* kilograms anywhere. Even one atom difference makes them not the same.

But (you say) we can use symbols to get as accurate as we like, can't we? After all, numbers are symbols and I can have a very precise number, like pi which is the ratio of the circumference of a circle to its diameter. pi is 3.14159265. Well, not exactly, because I used only 10 symbols (including the decimal), so I could have used more symbols:

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3.141592653589793238462643383279502884197169399375105820974944592307816406286  
208998628034825342117067982148086513282306647093844609550582231725359408128481  
117450284102701938521105559644622948954930381964428810975665933446128475648233  
786783165271201909145648566923460348610454326648213393607260249141273724587006  
606315588174881520920962829254091715364367892590360011330530548820466521384146  
951941511609433057270365759591953092186117381932611793105118548074462379962749  
567351885752724891227938183011949129833673362440656643086021394946395224737190  
702179860943702770539217176293176752384674818467669405132000568127145263560827  
785771342757789609173637178721468440901224953430146549585371050792279689258923  
542019956112129021960864034418159813629774771309960518707211349999998372978049  
951059731732816096318595024459455346908302642522308253344685035261931188171010  
003137838752886587533208381420617177669147303598253490428755468731159562863882

353787593751957781857780532171226806613001927876611195909216420198938095257201  
 065485863278865936153381827968230301952035301852968995773622599413891249721775  
 283479131515574857242454150695950829533116861727855889075098381754637464939319  
 255060400927701671139009848824012858361603563707660104710181942955596198946767  
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Except that pi goes on forever. So I cannot show pi exactly with symbols unless I plan to be talking for a very long time. Ok (you say), but what about colors? Well (I tell you), each color has a frequency. Visible light is just radio waves at very high frequencies like 500,000,000,000,000 Hz (Hz = Hertz, which is cycles per second; a measure of frequency named after Mr. Hertz who admired frequency in all things). Each color (except magenta, white, brown, and other mixtures) has a particular frequency. Unicorn’s mother’s eyes have exactly one frequency (well, we don’t really know that but let’s assume Unicorn is right; she is a rainbow expert after all). No matter how many symbols we use, we will never get the exact frequency. Why is that? Because real numbers (like pi) go on forever. They don’t have to, but most of them do.

There are numbers that don’t go on forever, or that go on in predictable ways. These are called “rational” numbers as opposed to “irrational numbers” (who would want an irrational number anyway?). Rational numbers are things like 0.2 which is 1/5, or 0.33333.... which is 1/3. Basically anything that is one integer divided by another. Rational numbers can be described precisely with two integers (the numerator and the denominator). Even if the integers are very big, I can write them down because sooner or later, every integer comes to an end. That would be fine, except that there just aren’t very many rational numbers compared to irrational numbers. There are infinitely many irrational numbers between any pair of rational numbers. In other words, a whole lot more of the never-ending numbers than of the ending or predictable numbers. Almost all real numbers are irrational. Hopefully the same is not true of people. Or goats.

What do I mean by “almost all real numbers are irrational”? After all, aren’t there infinitely many numbers? Seems rational to expect there should be plenty of each type to go around. Mathematicians talk about different kinds of infinity. There are infinitely many integers 0, 1, 2, 3 ...., and there are infinitely many rational numbers 1/2, 1/3, 1/4, ...., and there are infinitely many irrational numbers pi, 2pi, 3pi, 4pi, .... but there are different sizes of infinity. Whoa! That’s weird. Yup. Let me explain. We say that integers are “countably infinite” because we can count them. It might take forever, like counting the fingers and toes on a millipede, or a gazillionipide, or an infinitipede, but wherever you got to, you would know that you hadn’t missed any numbers so far. If I count to 1000, I know that I have named all the numbers less than 1000. If I count to 1,000,000 then I know that I have named all the numbers less than that. It turns out that the rational numbers (fractions; one integer divided by another integer, like 1/2, 2/3, 45/54) are also countable, because I can assign an integer to each one. How do I do that? Make a big table of all the rational numbers like this:

|     |     |     |     |          |
|-----|-----|-----|-----|----------|
| 1   | 2   | 3   | 4   | 5....    |
| 2   | 2/2 | 3/2 | 4/2 | 5/2 .... |
| 3   | 2/3 | 3/3 | 4/3 | 5/3 ...  |
| 4   | 2/4 | 3/4 | 4/4 | 5/4 ...  |
| 5   | 2/5 | 3/5 | 4/5 | 5/5 ...  |
| ... |     |     |     |          |

the table clearly has all the rational numbers in it if we go on forever (I hope you like fractions; there are a lot of them). But how can we count them and make sure we haven’t missed any? We do it by zig-zagging diagonally. So we count: 1, [2, 2], [3, 2/2, 3], [4, 3/2, 2/3, 4], [5, 4/2, 3/3, 2/4, 5], etc... Of course we will count the same number more than once (1 appears over

and over on the diagonal) but we won't miss any numbers. By counting, we assign an integer to each of the rational numbers. Because there is a 1:1 correspondence between integers (which are countable by definition) and rational numbers, we say that the rational numbers are countable. (If fractions scare you, you can rest assured that there are just as many fractions as integers so they don't have an advantage, at least by counting.) This is great, because we can always write rational numbers as a fraction, and the fraction is a ratio of two symbols, so " $x/y$ " is a perfectly good symbol for numbers, even if there are infinitely many. And for any rational number we choose, " $x/y$ " has a finite number of digits in the numerator and a finite number in the denominator. We can write them all down with a finite number of symbols. Just like writing a sentence with a finite number of words. All rational numbers come to an end. All sentences come to an end, no matter how long they are. And we can count all the rational numbers, and we can count all the sentences.

So what about the irrational numbers? Not so fast. Counting won't work here. Irrational numbers *cannot* be written as fractions with integer numerator and denominator. None of them. Let's just look at a single irrational number, pi. pi goes on forever, and there are no repeated patterns. A fraction such as  $1/3 = 0.33333\dots$  can go on forever, but it repeats so there really are only a finite number of different symbols, and we could write "0.33repeat" to indicate this if we wanted to. But pi goes on forever and never repeats. OK, you say. We will let pi be pi, but that's only one irrational number, right? Sure, but what if I take the first digit of pi and change it to a 2 instead of a 3, so we have  $\text{notpi} = 2.14159\dots$  instead of  $\text{pi} = 3.14159\dots$ .  $\text{notpi}$  goes on forever just like pi, but it's not pi. OK, now we have two of them. But I can change *any* of the digits in pi and get a new number that goes on forever. And there are infinitely many digits. No matter which one I change, I have a new irrational number. So suppose that I had somehow tried to count all the irrational numbers, and I've gotten all the way to pi after counting N of them. My evil friend Evil Eve could change just one digit on one of the N numbers, and now there is one I missed! OK, I say to Evil Eve, I missed one so I'll call that N+1. But then the evil thing does it again. N+2. Then again. uh oh. No matter how many numbers I count, I have always missed an infinite number of them! If I try to make a 1:1 match between integers and irrational numbers, I always miss an infinite number. Even before I get to the first one. So the irrational numbers are not just infinite, they are uncountably infinite. I can't count them. Which means there are more irrational numbers than rational numbers because I can count the rational numbers. In fact, there is an uncountable number of irrational numbers between any pair of rational numbers, no matter how close the pair is.

Wait, what? Ok, let's try again. When there are a lot of things, we assume we can count them. Bugs, flower petals, toothpicks. But the irrationally annoying thing about irrational numbers is that you can't count them. Suppose you try, and you say 1,2,3.... lining them all up as you go. Just when you think you are done, you realize that there is an infinite (uncountably infinite) number of irrational numbers between the ones you chose as number 1 and 2, and between number 2 and 3, and so on. So now you try to line up the numbers between the chosen 1 and 2, but between each of those there is an uncountably infinite number of new numbers that don't have positions in line yet. Not only won't you finish, you won't even start, because if you line up the numbers you cannot determine who is the second number in line, let alone the 10th. You are now in boarding group Z and there is no way you are going to make it onto the plane to Omaha. Ever.

Why am I telling you this? Because numbers are simple examples of symbols for things. Even in the perfect world of mathematics, symbols have their limits. Imagine if you needed a very very long sentence to describe a picture. That's fine, because sooner or later even an unnecessarily lengthy sentence with multiple dependent clauses each of which could be part of a compound clause or part of a clause with further dependencies, none of which may be important to the meaning but nevertheless they are there, will come to that final point of



punctuation, familiar to speakers of all Western languages, in which a single dot, a point with no dimension at all, fittingly if somewhat ironically, for irony is often the point of such a point, known as a period which by grammatical necessity signals the termination of what was begun so long before. In other words, it ends. Just like a rational number, for even an infinitely repeating rational number could end with "... and so on" and thus terminate. But the real world is not rational. It's mostly irrational. Which means that real things do not end. A perfect description, whether numerical or grammatical, will never end and never repeat. Because reality can always be described in finer and finer detail, and it is not in fact "turtles all the way down" but instead does not repeat, at least not exactly.

With rational numbers eventually every number comes to an end. With grammatical sentences, eventually every sentence comes to an end. You can count the number of possible sentences of any length just like you can count the number of possible rational numbers made from any two integers less than a certain length. But irrational numbers never end. Their length is always infinite. You cannot count them. How many infinitely long sentences without repeats are there? You will never know because you will never finish reading the first one, just like you will never get to the end of pi. Irrational numbers are not just infinite, they are "uncountably infinite". But sentences are countable, because there are only so many words and I can (in theory) count all the possible sentences that have N words or fewer, for any length N, just like I can count the number of fractions  $x/y$  where x and y are integers less than N.

The rational numbers are like tiny disconnected islands floating in a sea of irrationality. Not even islands, because the island itself is full of irrational holes. If you add up the land area of all the tiny dot-islands, you find out that it is zero! The rational numbers are so few and far between, that they don't take any space at all. Each one is a point, and points have size zero and no matter how many points of size zero you have all you get is an area of zero. All the real estate is owned by the irrational numbers. Rational will always be outvoted by an infinitely large irrational homeowner's association. Not good for the rational real estate market. What a sad day for rationality it was when the mathematicians figured this out. Sentences and groups of symbols are just a set of infinitely small unconnected islands in a huge sea of irrational reality.

So just like finite-length sentences, the rational numbers are not a good description of reality. Maybe Unicorn is right. But maybe we can fix this by letting them be an approximation of reality. Here is the *deus ex machina* of the rational numbers: The rational numbers are dense in the real numbers. Uh huh. What does that mean? It means that for any real number, you can find a rational number as close as you like. Play the game like this. Evil Eve chooses an irrational number, say pi. But to play, she also has to choose a distance, say 0.01. Then I have to find a rational number that is within that distance of pi. Easy-peezy: 3.14. Or 3.15. Because we know pi is between 3.14 and 3.15. Now Evil Eve is angry. It's not a pretty sight. She says. Ok, make it 0.0001. I say either 3.1415 or 3.1416. Oh she is hopping mad now. Really hopping. But she will always lose this game. I cannot tell her pi exactly, but if she picks any distance, no matter how small (but it can't be zero) I can always find a rational number. So the real-estate may be zero, but it's everywhere. Think of the rational numbers like cellphone towers. Not very big, but you are (hopefully) sufficiently close to at least one. If the signal is bad, you can always find another place to put a tower. It may never be exactly where you are standing, but it's close enough.

So for symbols: maybe we can say that sentences are dense in the set of all things we want to talk about. So long as we can choose how accurate we want to be, we can always end the sentence. "Unicorn" includes unicorns with any eye color. "Unicorn with blue eyes" includes unicorns with any eye color that is better described as blue than green or brown. "Unicorn with blue eyes and a pink tail" is even more specific. Just like saying that pi is close to 3. Or

close to 3.1. Or close to 3.1416. It's good enough for what we need to do. Sometimes the perfect is the enemy of the good. Since perfection is often irrational.

Etymological side note: the word "rational" in mathematics comes from "ratio" meaning the ratio of some integer  $x$  to some integer  $y$ , or the fraction  $x/y$ . Ratio-nal. The opposite should have been non-Ratio-nal instead of ir-Ration-al, but that would be too rational. The other use of "rational", as in "a rational argument" has various origins but some think is a mis-spelling of the French word "raisonable", based on the root "raison" which means reason. So we use "rational" when we mean "reasonable" but that's the (Lamarckian) evolution of Language.

Now back to rational numbers. We fix the zero-land-area problem by using approximation. Suppose I just have integers 0, 1, 2, 3 .... I can say that "3 means exactly 3.0000...", with an infinite number of zeros. "3" is a point, and points have no area. A piece of space-dust landing randomly on the real numbers has a zero probability of hitting one of the zero-area rational islands, just as you have a zero chance of standing on top of a cellphone tower while making a phone call, or a lizard has a zero chance of looking exactly exactly like a digital photograph of itself, no matter how good the quality. If I choose an irrational number at random, it might be very close to 3 but it will never be exactly 3. If I have a ruler marked 0cm, 1cm, 2cm, 3cm, etc and I throw a dart at the ruler, it will never land on exactly 3.000000.... . Because "exactly 3" has no width. But the way we fix this problem is we say, "what I really mean by 3 is anything from 2.5 to just a bit less than 3.5". Now if I throw a dart, I might actually land closer to 3 than to anything else. I can use this fix for all the rational numbers, because even though there are an infinite number of irrational numbers between each rational number, there is always a rational number that is as close as we want it to be, even though this annoys Evil Eve. And we do the same for words. Even though a finite-length group of symbols such as "green lizard with six toes" is really only a point in word-space, we can take it to mean "green lizard with more than 5 and 1/2 toes and less than 6 and 1/2 toes" and now there is an island of lizards that match. And we do this all the time. "Blue" means anything that looks more blue than any other color (even if not everyone agrees). We build in the imprecision so that we can describe things we see without having to describe everything about them (which would go on forever).

So back to Unicorn's concern. No matter how many words (or symbols) you use, you can never tell the exact color of her mother's eyes. You can't even create a precise category such as "all unicorns with eye colors with a higher frequency than mom's eyes". Symbols are always imprecise when they describe things in the real world. That's because sooner or later, you have to stop the sentence, or stop figuring out digits of pi, or just say the number will be approximate. Once you do that, you have left open an (uncountably) infinite number of possible numbers. For example, suppose I say that pi is between 3 and 4. That leaves open a lot of options. Suppose I want to be more precise and say pi is between 3.1 and 3.2. Still a lot of options. OK, how about 3.1415926 and 3.1415927. Still just as (uncountably) many options. Suppose we play a different game with Evil Eve. She says, "I'm thinking of an irrational number. Can you guess what it is?". I guess 3.14? Eve evilly says "close, but it's bigger than that". So I say 3.141 (I'm guessing it's pi, but I'm not sure). She says, "bigger than that", so I say 3.1416 and Eve even more evilly says "smaller", and so on we go. I will never win. Because I didn't force Eve to tell me how close I have to get to win. I can only win if I get it exactly, and that will never happen. Even if I guess and say "is it pi?" Eve could say, "actually, it's  $\pi + 0.00001$ " (ooh she is evil). Any time you write down (or say) a symbol, there are many, many, (uncountably) many possible things that are consistent with that symbol. Unless you are willing to write forever, but forever never ends so you can't really do that.

But (you ask), what about the word "pi"? Isn't that exactly the thing we want? Sure. it's perfect. Just like the number 3. Or the symbol for infinity. But it all fails as soon as we ask the question about something in the real world. For example, what is the ratio of the

circumference of this particular circle to its diameter? It's probably close to pi, but maybe not exact because it's not a perfect circle. And no matter how much precision we use to measure, there are always gaps in our ruler. You cannot measure the irrational world with a finite ruler.

But imprecision is a good thing, because it means that sentences eventually stop. And just like decimal numbers (3.1416) we can stop our sentence whenever we have reached as much precision as we want to have. For example, if we look at a picture of a unicorn and say "that's a picture of a unicorn", we have left options for other details in the picture (how big, what color, what shape) and collapsed all the possibilities into a single word "unicorn". You take a single unique example of a unicorn, and re-label it by the category. Now you are talking about lots of unicorns. No one likes to be categorized, least of all unicorns. If you use logic based on the category, you may be very wrong. For example, if you believe that all unicorns are bad-tempered, you will think that Constance is bad-tempered (which she is, but not always). This can get wigglier if you use more complex logic (a "Syllogism"). For example:

All unicorns have one horn.  
Constance is a unicorn.  
Therefore Constance has one horn.

This sounds reasonable, because just like  $2+2=4$ , it is just a relationship among symbols (words). But in the real world, Constance might be a unicorn but not have a horn (if it were damaged) or she might have more than one horn (a fashion statement among some of the more modern unicorns). OK (you say), the problem is the first statement "All unicorns have one horn", which might be false. But then what do we learn by calling Constance a unicorn? If that really says nothing about Constance (not even how many horns she can toot) then why say it at all? The real world is more like:

Some unicorns have one horn, although scientists disagree on this.  
Constance believes she is a unicorn, although it is unlikely since unicorns are rare.  
Therefore Constance might have one horn. Or not. Depends on what a "horn" is.

So after all that, maybe Lizard and Unicorn are correct, and words really aren't worth talking about. Except for the obvious irony that to talk about them at all we need words. To have a build-a-talking-brain class you need words. So there must be something useful in symbols, right?

The advantage is that sometimes precision is too much of a good thing. Goat's pound cake will taste just fine if I have 1.1 pounds of sugar and 0.9 pounds of flour. Why waste time traveling to Paris to compare to the perfect kilogram, and then dividing by approximately 2.2 (to convert the perfect kilogram to an imperfect pound)? And instead of listing an infinite number of digits I can just say "pi" and everyone knows what I mean (unless I'm in a bakery). And sometimes, blue is blue, and a unicorn is a unicorn, and an elephant is an elephant. And sometimes, all unicorns do have horns, or bad tempers, and that can be helpful information because it generalizes to other unicorns.



And by allowing imprecision, we gain something amazing: fast communication! The word “unicorn” can be pronounced in under a half second (approximately) and yet provide a tremendous amount of information to the listener, including the number of horns, expertise in rainbows, and a love for cute fuzzy bunnies and rock candy. Even if all of those things don’t describe Constance, maybe we just want to talk about unicorns in general anyway. And then I can put words together, to say things like “the tall unicorn with the fuzzy tail and blue eyes that is trying to learn to throw darts was able to warm the heart of the unhappy bird we saw in the playground on Tuesday.” It’s still very imprecise (which playground?) but it’s good enough. You could even write that on a piece of paper and give it to Cat who will transport it across town on his bicycle. The more words I use, the more precise I can be. The fewer words I use, the more concise I can be. So I have to choose whether to be precise or concise. But keep in mind that if you choose precision over concision, you may induce somnolence in your listeners due to very long sentences. If you choose concision over precision, you may induce confusion, or give the wrong idea entirely (“last night I saw an elephant in my pyjamas”). So your choice is somnolence vs. confusion. Biochemists, philosophers, and mathematicians can induce both. Hopefully there’s a happy median.

Symbols (words or numbers) help with being concise. As an example, consider the way numbers work. If you want to tell someone that you have one hundred and twenty-seven beans, you could show them a picture of the beans which they could count out on the fingers and toes of themselves and 12 friends, or you could just use “127”. With just three words, you can write or say any number between 000 and 999. That is the *combinatorial* power of words (and numbers): for each of ten possible numbers in the hundreds place, you have ten possible numbers in the tens place, and ten possible numbers in the ones place, which gives  $10 \times 10 \times 10 = 1000$  possibilities, or 10 to the third power. The more places you have, the more numbers you have. If you have 4 places you have 10 to the fourth power, and so on. By the time you have 90 places you can count every atom in the universe. Now do the same thing for words. Most of us know about 30,000 different words (Owl probably knows more). The rules of grammar don’t let you put any old word anywhere in a sentence, so let’s assume that really there are only about 1000 possible word possibilities for each position of a sentence. Then there are enough 30-word sentences to write one on each atom in the universe. And really, if you had a billion monkeys typing randomly and were hoping that one of them would sooner or later by chance type out the script to “Hamlet”, you can be pretty sure they would all run out of ink molecules long before they got past act 1. And they are just as likely to write out Aristophanes “The Frogs” (in Greek) before they get to Act 1 of Hamlet. So the point is that using words (symbols) really gives you a lot of possibilities. More than the number of atoms in

the universe. And that means you don't need that many words to express a very large number of ideas, just as you don't need that many numerals (0-9) to express a very large number of (rational) numbers. Symbols are very efficient.

You don't even need to use particularly long sentences to express very complex concepts. This really helps when you are a bicycle messenger. Or succinct. Or pressed (for time). Cat is named after Claude Shannon, who created the field of *Information Theory*. Information theory tries to figure out how to get the most information from one place to another when there is noise, perhaps on a radio station, or because you bought the budget cellphone from "fell-off-a-truck Ralph". Human speech is actually very slow. So is human writing and typing. Nowhere near as fast as a 1200 baud modem (remember those?) or gigabit ethernet. So one of the amazing things about human language is that it allows transmission of detailed information over the slow speech channel. Even with lots of background noise like during a party or using a poor-quality cellphone or in a thick accent.

But (you ask), if I want to actually transmit *speech* then don't I need a good quality internet connection? True, but it depends what is the information you are trying to transmit. If you just want to say "have you seen the cat" you only need 21 characters which at 1200 bits per second (about 8 bits per character) requires less than two tenths of a second, whereas a recording of you saying that would take 1-2 seconds depending on whether you grew up in New York or New Orleans. But if you want to sing the refrain to the song "Have You Seen the Cat" in order to entertain your cat-less listener on the other end of the phone, then you had better send all 2 seconds of data, ideally at 192,000 bits per second if you want good music quality. This is the same concept as typing the word "blue" or sending a picture of the exact blue you want. The point is that the symbols give the minimum amount of information you need to transmit the symbolic concept at the level of precision you choose. That doesn't stop you from transmitting using more information (music, video, calligraphy) but you can be more concise if you want. You just can't get any more concise than the words themselves (unless you can predict the words in advance, in which case you don't need to transmit anything at all, or perhaps just the first word in the sentence, or rmv th vwls). As I said above, the important point is that symbols transmit information that is concise but imprecise. More concise requires less precise. More precise requires less concise. Doesn't matter how fast you talk, you have to trade off precision for concision. And of course Cat would prefer conciseness.

Speaking of cats, Cat pointed out another very important thing about written words, that also applies to spoken words. Symbols are *noise tolerant*. This means that the symbolic word "blue", even with mud on it, is still probably readable as "blue". Whereas a non-symbolic photograph with mud on it might not be recognizable as blue, or at least not the same blue. This is also true for words; you are unlikely to confuse hearing "blue" with "green", although you might confuse it with "shoe", "crew", or "new". But that wouldn't happen because in context it wouldn't make sense to say "I want draperies in light shoe to match my mother's eyes." (Unless your mother is in fact an old lady living in a shoe; I know an old lady like that.) In fact, we can use words to increase the disambiguity, by spelling out names using military radio-speak "alpha bravo charlie delta...." or "bravo lima uniform echo". The point is that symbols have a sort of a gravity around them so that if you don't get too far from one symbol, it sucks you back toward it. Things can get close ("that's a blue brew, bud"), but there's always at least some space (unless you can't bear bare bears).

The idea of space between symbols is an essential reason why electronic computers are possible. You may be aware that computers can come in two flavors: digital and analog. An analog computer can add  $1 + 1$  by putting 1 volt on one wire and 1 volt on another wire and combining them using some electronic parts to get 2 volts out the other end. A digital computer can add  $1 + 1$  by representing 1 using the binary digit "1" which is represented by

any voltage greater than 1.5volts (usually at least 3 volts), and the binary digit “0” by any voltage less than 1.5 volts (usually at most 0.5 volts), and then doing some electronic magic to get “10” (the binary representation of the number “2”) out the other end. In volts, the digital computer adds 3 volts (which means “1”) plus 3 volts (which means “1”) to generate two wires, one of which has 3 volts and one of which has 0.5 volts, which means “10” which means “2”. The advantage of digital is that when the batteries run down on the analog computer the voltages drop and you get  $0.8 + 0.8 = 1.6$  which is the wrong answer. When the batteries run down on the digital computer, you get 2.5volts plus 2.5 volts gives two outputs of 2.5 volts and 0.3 volts which still gives “10” and means the correct output of “2”. In the same way, if we add noise to the wires, or use cheap components, the digital computer doesn’t really care about small changes in voltage, whereas the analog computer will always get the wrong answer unless everything is perfect. (To be fair, analog computers have some other advantages but they don’t make my point so I’m not going to tell you. Author’s prerogative.)

But hopefully you can see that symbols (ie: words) share the same advantages of digital computers: noise tolerant, fast communication over poor quality channels, lots of possible things you can say. They also share the disadvantages of digital computers, which is that it takes a lot of data to communicate things that aren’t symbols (like pictures or raw sound). Think of how long it takes your phone to download grandma’s high-quality picture of her wild birthday party in Cancun compared to the time needed to download her accompanying text saying how disappointed she is that you forgot to call her on her birthday. And no matter how good quality your downloaded picture, there is always a higher-quality picture, and nothing is ever as good as being there for the real thing (ask your grandma).

The word “digital” comes from the latin “digitalis” which means finger. The same word is used for the plant *foxford purple* and the chemical *digitalis* that can be extracted from foxglove and which makes your heart pump harder. So in one of Nature’s wonderful etymological ironies, digital is at the heart of all life. But this book is about brains, not hearts, so we will keep our fingers pointed at the heart of the current subject matter. Which is the idea that you can count (“digitally”) on your fingers, by putting the right number of fingers up or down. Two fingers up means 2, three fingers up means 3, etc. Unlike Arabic, Roman, or binary numerals, which two fingers are raised does not change the value. In the binary system, for instance, the first finger indicates the presence of a 1, whereas the second is a 2, and the combination of the two would be a 3. This is too annoying for most people; while we may be digital we do not use the place-value system for telling the maitre’d how big a table we need.

But imagine an analog-finger system, in which numbers were determined by *how far* you raised each finger. Maybe a fist is zero, pinky just a little bit up is 1, a little bit more is 2, and all the way up is 10. Then you could keep count just with your pinky while never letting go of your teacup. This sounds good, until you try to do it. First of all, it’s hard to hold your finger exactly 3/10 of the way up. Second, it’s hard for someone else to know if that’s what you are doing. What if you have arthritis? Are wearing gloves? Are you hanging from a branch upside-down? Are you being all shook up? Are you in congestive heart failure? So now we are back to noise sensitivity, and we lose accuracy. For the most part, someone holding up 3 fingers will be interpretable as indicating 3 even if arthritic, gloved, inverted, shaken, or congestive (in the last case, perhaps your 3 fingers are indicating the number of digitalis pills you would like to be given).

The thing that makes something binary is that it is either ON or OFF. Like a finger being UP or DOWN. Or eyes being BLUE or NOT BLUE. You can qualify if you like, but then your eyes might be [A BIT BLUE] or NOT [A BIT BLUE]. It’s still a binary choice. The binary number system uses a 0 or 1, and that can mean a thing or a number such as “101” which is five in binary (“1 if by land, 10 if by sea”). But binary just means there are two possibilities. It’s ok to

have a *ternary* number system which is base 3. That uses 0, 1, and 2 as the digits, and if you used your hands you might have a finger down, halfway up, or all the way up. The number “101” is ten in ternary. If you wanted 3 digitalis pills, you could put one finger all the way up and one finger halfway up which would be a  $2+1=3$ . Since fingers don’t use place values (such as the ones, threes, and nines place), you could use any two fingers for this. Which might be helpful if you need 3 tablets but only have 2 fingers.

Now wait a minute (you say); if I can do numbers in binary, ternary, quaternary, or gazillionary, and all numbers are symbols, then symbols don’t have to be binary. They could be ternary, quaternary, or gazillionary. And you are correct. What makes something a symbol is not the number system; it’s whether it is a category that contains many things. The number 3 could be used for 3 apples, 3 unicorns, or 3 philosophical theories. There is also a space between the symbols. Many things can fit between a 3 and a 4. Even if we use the decimal system to create a rational number (perhaps 3.5), there are an (uncountably) infinite number of irrational numbers between any two rational numbers. (Yes, I came back to that. You knew I would.) The same is true for words. The word “elephant” includes big elephants, little elephants, baby elephants, and elephant seals. Once you say it, you have gone all binary on “elephant” and there are lots of possibilities. Which may be exactly what you want. You may be talking about all of them. Or at least most of them.

The rational numbers are the only ones we can write down, because irrational numbers go on forever. And there are many more irrational numbers than rational numbers, so the chance that the size of something in Nature would be rational is very very small (essentially zero). However, one of the most amazing things about symbols is we may still be able to describe *certain* irrational numbers by how they behave. For instance,  $\pi$  is the ratio of the circumference to the diameter of a circle, but of course the circle would have to be perfect and we don’t have one of those and even if we did we couldn’t measure the ratio without making at least a teensy error. Many people and computers spend their spare time trying to figure out many digits of pi, but they always have to stop eventually, which leaves an infinite number of digits unfound. But that doesn’t mean we can’t use pi. Maybe you noticed that “pi” is a symbol for pi. The symbol represents the exact thing, even if we don’t know what it is. Just the way “unicorn” is a symbol for something we are unlikely to ever see, “pi” is a symbol for something we cannot ever see completely. But we can still do math with it.  $\pi+\pi=2\pi$  is a true statement. If I double the diameter of a circle, I double its circumference. If I add one to the diameter of a circle, I add pi to its circumference. These are all true (and possibly useful) statements, even without knowing the exact value of pi, or whether I added one centimeter, one inch, or one furlong. Hence the potential for symbols to manipulate irrational or even unknown numbers. And something either is or is not pi; so “being pi” or “not being pi” is an ON or OFF binary decision, even if we cannot quite tell in the real world.

What makes something a symbol is that it is *discrete*. In math, the opposite of discrete is *continuous*, and it would be a mathematical indiscretion to publicly proclaim otherwise. Symbols are discrete. Even when posted on billboards. The real world is continuous, at least as far as any of us can see (“what about quantum mechanics”, you ask? Be careful what you ask for. It won’t make any difference at the level of how your brain works anyway.) There are a lot of symbols, but they are still discrete, in the sense that there is space or a gap between the symbols. The written or spoken symbols “Blue” and “Green” do not look like each other, even if the two parts of the spectrum they refer to are right up against each other. There is a gap between the written or spoken words “blue” and “green” which is all the squiggles that are not either of those words, or all the weird sounds that aren’t one or the other, or all the different ways they could be pronounced. The script for “Hamlet” is not the same as the script for “Hamlet” written with a soliloquy that starts with “Two B or not two B.” Just four letters apart, but it loses impact if Hamlet is just trying to figure out the correct spelling of “flabbergasted”.

Continuous things irrationally can take on any value, and can move smoothly from one value to the next one (so long as no one is counting). A rock can have any weight. A stove can have any temperature. Eyes can have any color. The weight, temperature, and color can change smoothly with erosion, gas pressure, or age. But if you want to tell someone the *exact* temperature, weight, or color, it will take you an uncountably infinite amount of time. And if you want to tell them an *approximate* (finite precision) temperature, weight, or color, then there will always be imprecision in the temperature because of the gap between the symbols, in the sense that the symbols do not flow smoothly one onto the other, which is a good thing because we need to tell words apart.

So where am I going with all this? The idea is that words are symbols, symbols are digital, and digital things can never be perfectly precise descriptions of analog reality. But we can still describe height (I, for example, am 5 foot 11 inches or 180cm tall. Approximately.) The key is that we use *finite precision*. When I say I am 5 foot 11 inches, I mean that I am somewhere between 5 foot 10.5 inches and 5 foot 11.5 inches. Or maybe I could be 5 foot 11.999 inches, but I just don't want to claim that I'm 6 feet tall because that would not be true. Every measurement has finite precision. A thermometer can only measure in whole degrees, or tenths of a degree, or hundredths of a degree. There is always a limit. A ruler can only measure in cm, mm, or multiples of the wavelength of light. This is because you have to convert the output of the measurement to symbols; the numbers that actually indicate the weight, temperature, or length. And since, like Owl, you want to be rational about it, you can only use rational numbers so that you don't have to take an infinite amount of time (and ink) to answer how tall you are.

Despite Unicorn's protests, we do the same with words. The wavelength of blue light is between about 430 and 500nanometers. The wavelength of cyan (blue-green) light is 500-520nm, and the wavelength of green light is 520-565nm. But does light change from blue to cyan at exactly 500nm? Is 499nm blue and 501nm cyan? Of course not. But we have to draw a line somewhere. Someone else (particularly if they are colorblind or wearing rose-colored glasses) might put the line somewhere else. We divide the spectrum of visible light into approximately 7 regions: red, orange, yellow, green, blue, indigo, violet. Or perhaps red, orange, yellow, green, cyan, blue, violet. Or we could label light by the wavelength to two digits of precision where blue is 430, 440, 450, 460, 470, 480, and 490nm (approximately). Or we could label light with the wavelength to 3 digits of precision: 430, 431, 432, .... 500nm.

We can do the same with other categories. Consider "mammal". Most mammals we know do not lay eggs. Except the duck-billed platypus (and a few others). So you could be excused for excluding platypus from our mammalian friends. They do, after all have flat feet like a duck (platus = flat, pous = foot), although my cousin Ernie has flat feet and even if this makes him a platypus I don't think he lays eggs. (If a duck ever invoiced Ernie for egg-laying services, then Ernie would also be duck-billed.) But scientists decided that platypi (platypuses? platypodia?) are mammals, because mammals give milk to their young (and latte's to their less young) and platypi do this, despite looking like ducks and having poison in their feet (they are from Australia, where many cute fuzzy things have the ability to kill you). Every symbol tells us a category, and every category has boundaries, and boundaries are in the eye of the beholder, although sometimes the beholders decide to agree. But boundaries imply a middle, and there is almost always more than one thing in the middle. "Lizard" can be any lizard. "Green Lizard" is a smaller set of lizards, but still a set. "Leonardo the Lizard" is the set of all lizards named Leonardo, although it is possible to be more precise and say "the Lizard in this room" and then we know which one it is, as long as we are sure there is only one lizard in the room.

You can use more words and make the category smaller (a young coffee-colored duck-billed platypus named Peter on his way to buy arch supports for his feet), but it's still a category



because there could be more than one platypus that fits. Just like you can use more digits of precision in a number but at some point there are (uncountably) many numbers that will fit the bill (the patient's temperature is somewhere between 98.65238 and 98.65239 degrees). OK, you get it. Symbols (words and numbers) are concise but to get concision you have to lose precision. And often that's just fine.

But here's the really cool thing about symbols: you can make stuff up. What about a dolphin-billed platypus? Or a blue-throated kangaroo? Or a flying donkey? Or zombies? Since there are enough word combinations for more than every molecule in the universe, we can combine words to describe many, many things that can never happen. We can also describe things we haven't seen but that might be real, such as the Aurora Borealis or political compromise. We can describe things we know aren't real (a flying elephant) and things that are real but maybe not that big (an ego the size of a house). We can describe things that aren't real but that we intend to make real (flat-screen displays, wireless telephones, a cure for bacterial infections), and we can plan and execute complicated actions by groups of people (everyone in an approximately red shirt on this side of the field; everyone in an approximately blue shirt on that side of the field; now slowly approach the groundhog from both sides). Symbols can describe things at different levels of abstraction, moving farther or closer to the analog world (this duck-billed platypus; all platypi; all mammals; all poisonous things; all dangerous things; all conceptual taxonomies of evolution). Because we can combine words to indicate different concepts, we can say that symbols are "combinatoric".

So to give away part of the punchline of this book, I'm going to claim that language is not the most important part of being verbal; it's the use of symbols. Symbols allow us to categorize, to perform logic and mathematics, to describe or imagine things (unicorns) that we have never seen, to dream up things (flying cars) that have never been built, to talk about groups that don't exist (teenage non-users of social media), and to create cultures, governments, and job descriptions within large organizations. Symbols are much more than just communication. We can use symbols even when we are by ourselves, to figure out why something happened, to plan tomorrow's dinner, to balance a checkbook (remember that?), to design a car, or to decide whether to buy a car. Symbols allow us to function discretely in a continuous world, and they give us a huge advantage over species that do not use symbols (which is pretty much all the other ones, including probably other species of the genus *Homo* all of whom we killed off or traded to alien species in exchange for the recipe for non-dairy milkshakes (well, that's one possibility and who's to say I'm wrong?)). And one of the most important uses of symbols is to describe tools, and categories of tools, and the principle behind how different tools work or how to build them, and to write these down and to teach them to anyone who is interested and to remember what we know and to spread knowledge that one person discovers by accident to everyone else who did not discover that knowledge. And another use is to organize people into groups and to assign tasks and to break down complex tasks into simple tasks and create joint projects and assembly lines and organizations and armies. Once you have symbols and you have language, you get everything else with it. All at once.

Discrete symbols let us have many different and flexible behaviors. There are so many possible combinations of symbols, that at any given time every human on the planet can be thinking and acting on a completely different set of symbols. Compare the behavior of a flock of birds on the beach to the behavior of a herd of humans on the beach. The humans are all over the place, opening sun umbrellas, putting on music, building sandcastles, surfing, sunbathing, whatever. The birds all stand facing the same direction. If one moves, they all move. If one flies off, they all fly off. And there's not much else they can do. Sit, stand, turn, walk, fly, tweet. The difference in the brain from one bird to another is limited to the differences in what they see and feel. They (probably) do not have differences in imagination, belief, thought, and reason, except in Hitchcock movies. They see the analog world, and there

is only one analog world, and they all see the same one, which is the only thing that drives their behavior.

Here is a summary:

Symbols:

1. Binary on/off: You either say it or you don't.
2. Discrete digital: there is "gap" between different symbols; no blended words
3. Noise tolerant: even with dirt on it, the word "blue" does not change its meaning
4. Combinatoric and countable: can imagine or say many things by combining symbols, but can always count the possibilities.
5. Imprecise: can only approximate reality, so always indicate a range or more than one thing
6. Concise: writing is very efficient for small messages
7. Encodable: can use many ways to transmit: sound, writing, sign language

Non-symbols:

1. not Binary: more/brighter/louder changes the meaning
2. Continuous: no "gap"; can move smoothly between different things
3. Noise sensitive: dirt changes the color
4. Combinatoric but uncountable: can combine pictures or colors or numbers, but cannot count all the possibilities
5. Precise: each thing is exactly what it is. pi is pi, even if we cannot write it down
6. Not concise: can take infinitely long to write as a message
7. Not encodable: a picture is not the same as the thing itself

This book is about symbols. Language is what symbols look like when you try to communicate them to someone else. It is the imprint of symbols on the analog world. We use sound (something analog) to communicate symbols (something digital). Once something is discrete, it stays discrete. The symbol for "cat" leads to the sound "cat" which has a space between it and other sounds such as "cut", "cot", "bat", or "pat". "Verbal" is what we are when we use sound (or sign language, or writing) to communicate symbols. There are other forms of non-verbal communication of course, which are used to communicate emotion, emphasis, intention, anxiety, discomfort, etc. These are analog (unless we are using words to talk to a therapist) because the more emotional you are the more emotion you will express; it's analog. There is no boundary, it cannot be measured precisely, and there is no space between different representations.

If you are thinking I'm about to join team Owl, you are incorrect, wrong, and mistaken. Symbols have great power, but only when they are connected to the analog world. More about this later. Now, back to class.